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Bed forms in alluvial channels

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A solution is offered for the relative phase relationship of bed, depth and surface waves observed in alluvial channels, and is found to be in good agreement with observation in laboratory flumes. The solution does not depend on an assumption of a phase lag between the velocity and sediment movement. The emphasis of the paper is on the mechanics of bed-wave formation.

1. Introduction

For centuries man has been intrigued by the formation of ripple marks and dunes under flowing water, yet relatively little progress has been made in analytical formulation of this phenomenon. The analytical models are, broadly speaking, of two kinds; those which assume an ideal fluid (Exner 1925; Anderson 1953; Kennedy 1963), and others considering the one-dimensional flow over a wavy bed with constant coefficient of friction (Exner 1925; Henderson 1964; Reynolds 1965). Both these approaches assume that the sediment movement along the bed is a function of the mean velocity of flow. Cartwright (1959) introduced a phase lag between the velocity and sediment movement which was also used by Kennedy, together with this dependence on mean velocity of flow. It is the purpose of this note to show that neither model is in agreement with the observed mechanics of the bed-form formation, and to propose a partial solution in better agreement with the observation.

2. Observed mechanism

It has been shown (Raudkivi 1963) that the sediment movement is not continuous and that over a substantial part of the grain boundary the direction reverses. In general, the sediment movement depends on the wake formation in the lee of the bed form and on the combined effect of local turbulent agitation and local temporal mean drag. When the threshold of grain movement is passed a chance piling-up of grains creates a local disturbance. This disturbance of the plane grain boundary establishes an interface or surface of discontinuity in the flow, similar to that in flow past a negative step. In the shear flow of this interface the turbulence intensity is high and the grains are stirred up by the turbulent agitation where the interface reattaches itself to the grain boundary. From the reattachment point downstream the turbulent agitation decreases and also a boundary layer develops. The temporal mean shear, which provides the power for the transport of grains, is zero at the reattachment point and increases in the downstream direction; the reversal under the wake is usually of small intensity.

Because of the reduced agitation some of the material made mobile at the reattachment point cannot be supported and settles out as it passes downstream. This leads to a second disturbance, and so on. Here it is important to observe that the bed-form development propagates gradually downstream from the point of disturbance. It is not a process of amplification of a bed disturbance of given wavelength. Figure 1 (plate 1) shows the front of such a propagation and figure 2 (plate 1) shows the steady-state pattern. The ripple form travels by erosion of sand on the upstream face and by deposition in the lee of the ripple, so that each grain travels intermittently and is buried for periods. In the lee of each bed perturbation of the flat surface the forward transport of sediment stops and material accumulates until fluid drag over the crest limits the growth in height. Thus, over the initial flat-grain boundary, a surface perturbation causes the temporal mean boundary shear on the immediate downstream area to fall to a slightly negative value. At the same time the shear stress acting on the fluid passing over the wake is increased.

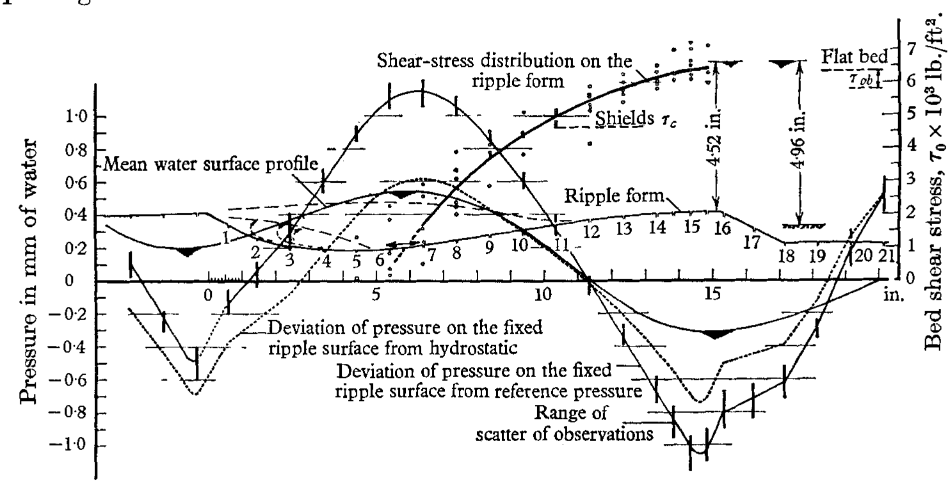

FIGURE 3. Ripple form, water surface profile. Distribution of pressure deviations and bed shear stress.

Figures 3 and 4 show results from measurement on an actual ripple form. The first important observation here is that entrainment and bed load transport take place at values of temporal mean shear stress τ_0 well below the value of Shields's (1936) critical shear stress τ_c which is reached at more than half way up the ripple face. The temporal mean shear stress is a maximum on the crest of the ripple and by observation appears to be equal to the mean shear for the same flow over the same sand bed in its initial flat condition. This flat-bed value of shear stress appears to govern the amount of sediment in transport.

It is also seen that the streamlines of the main flow (subcritical) are approximately sinusoidal, rising over the wakes and falling over the crests of the ripples. The main-flow lower boundary is formed by the tops of the wakes and the crests of the ripples and is out of phase with the actual sand boundary. Along the sand boundary there is a longitudinal force acting on the fluid which varies in magnitude with distance—an oscillating force which is not included in the analytical models—and the transport of grains is discontinuous with respect to the distance along the flow. Thus the equations of motion should include shear stress as a function of distance along the flow.

For the bed features to grow from an initial small disturbance to the steady-state pattern the applied mean shear stress on the bed must be high enough to effect sediment transport. Only then will the first disturbance grow. If this is not the case a local disturbance will decay. For example, a small hollow in the flat bed, when the average applied bed shear stress is below the threshold value, will lead to a sequence of bed forms similar to those shown in figure 1 (plate 1), but this will not grow or propagate (Raudkivi 1963).

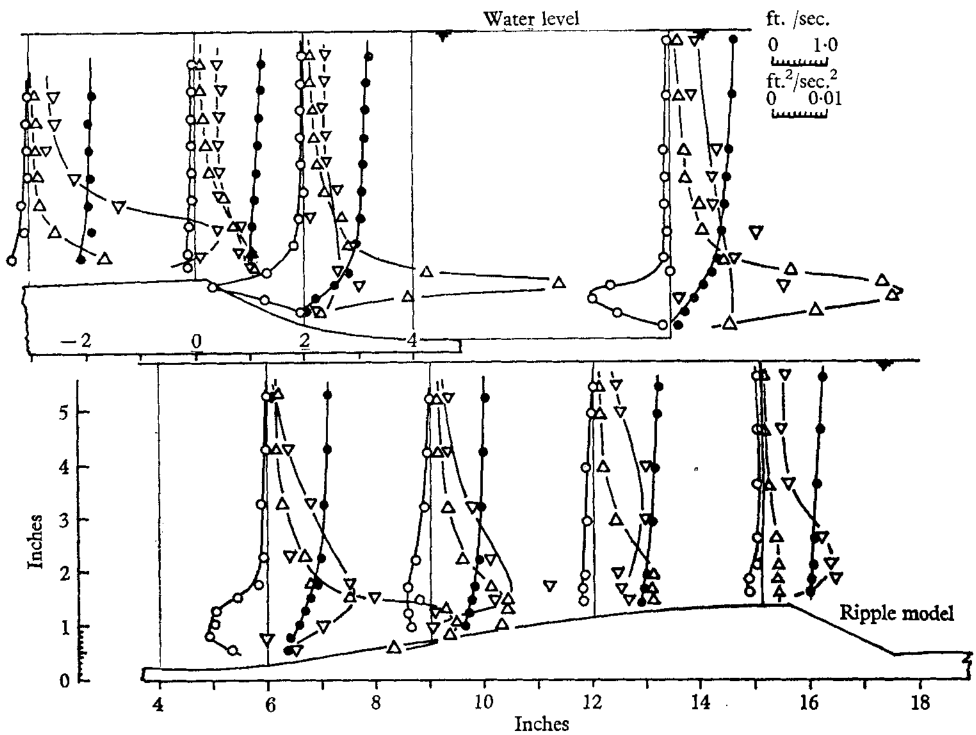

FIGURE 4. Profiles of mean velocity, longitudinal and transverse fluctuations, and turbulence shear along the ripple form. —●—●—, $\overline{U}$; —△—△—△, $\overline{u^2}$; —▽—▽—, $\overline{v^2}$; —○—○—, $\overline{uv}$.

The preceding discussion deals with sandy materials subject to a steady uni-directional flow of water. When the sediment in transport is coarse shingle the process discussed above of bed-form formation is greatly overshadowed by the strong turbulent agitation of the fluid flow and the associated impulsive forces on grains.

3. An approximate model

Considering a one-dimensional problem with the x-axis in the average bed surface, and assuming that the pressure is hydrostatic and the shear stress varies linearly through depth, then on neglecting $\partial u/\partial t$ one obtains

$$u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial}{\partial x}(\gamma m+\gamma\eta)-\frac{\tau}{\rho m}. \tag{1}$$

Here u is the velocity in x-direction, ρ is density of fluid, γ is specific weight of fluid, τ is the shear stress, m is the local depth of flow and η is the ordinate of the bed surface. Substituting from continuity ($mu = q$) and linearizing yields

$$\frac{m_0}{u_0}(1-Fr_0^2)\frac{\partial u^1}{\partial x} = \frac{\partial \eta^1}{\partial x}+\frac{\tau^1}{\gamma m_0}+\frac{S_0}{u_0}u^1, \tag{2}$$

where Fr_0 is the Froude number of unperturbed flow $= u_0/\sqrt{(gm_0)}$, and suffixes 0 and 1 refer to the unperturbed state and perturbation, respectively.

Assuming that the departure from the steady-state mean shear

$$\tau^1 = K\eta^1, \tag{3}$$

which is in keeping with the form discussed before, and

$$\eta^1 = A\,e^{ik(x-ct)}, \tag{4}$$

$$u^1 = Bu_0\,e^{ik(x-ct)}; \quad B = \alpha+i\beta. \tag{5}$$

The assumption of equation (3) has not yet been verified experimentally for super-critical flow, therefore the deductions for the super-critical range are speculative only. On substitution and separation of real and imaginary parts one obtains

$$\tan\theta = \frac{\beta}{\alpha} = \frac{-K(Fr_0^2-1)/\gamma S_0+1}{K/k\gamma m_0+km_0(Fr_0^2-1)/S_0}. \tag{6}$$

This shows that in subcritical flow the phase shift θ between the bed and the velocity wave may be in the first or second quadrant depending on the relative size of the terms in the denominator. In all cases β is positive but α may be positive or negative. Thus for long waves the wave-number k is small, $K/k\gamma m_0$ dominates, and the phase shift is in the first quadrant. The velocity wave and depth wave are related by continuity $[m^1 = -(m_0/u_0)u^1]$, so that the depth wave leads the bed wave by a phase angle $(\pi+\theta)$. The surface wave is given by the vector addition of the bed and depth waves. In this case the surface-wave angle would be in the third or fourth quadrant depending on the size of the two vectors and on angle θ.

For short waves and subcritical flow α is negative and the velocity wave is in the second quadrant. The depth wave will lead the bed wave by an angle greater than $\frac{3}{2}\pi$ and the surface wave will always be in the fourth quadrant, that is, close behind the bed wave.

In super-critical flow ($Fr > 1$) the denominator α is always positive. The numerator β may be positive or negative depending on the value of Fr. At values of Fr only slightly greater than one the numerator may still be positive and the surface wave may be in third or fourth quadrant. This is in keeping with the observation that at the limit of stability the surface waves break upstream of the crest of bed waves. For large values of Fr the numerator becomes negative and the surface wave will be in the first or second quadrant if not modified by the breaking of the surface waves. Such a situation could occur with long waves of small amplitude yielding a small negative value for $\tan\theta$ so that the bed and surface waves are approximately 90° out of phase.

Here two comments could be made. First, the analysis by Henderson, which assumed a constant friction coefficient, predicted the surface wave to be always in the fourth quadrant. Secondly, photographic evidence (Kennedy 1961) shows that the surface wave may be in the fourth or the first quadrant as predicted by this analysis.

Substitution of values pertaining to figure 3 in equation (6) yields a phase lag θ close to 180°, i.e. the surface and bed waves are nearly in phase, with the surface wave a little behind the bed wave. This agrees with the observation discussed earlier that the top of the wake and the crest of the ripple form the lower boundary and this is nearly in phase with the surface wave.

The problem of phase shift has to be considered relative to the actual lower boundary to the flow. It is seen that the actual lower boundary—the top of the wake and the crest region of the bed form—is nearly in phase with the surface wave. With increasing velocity the wake is 'filled in' by the rapidly moving sediment and the 'nearly-in-phase' picture emerges again. With long and flat bed waves the wake formation is insignificant; $\tan\theta$ and the surface wave amplitude are nearly zero. Also, at super-critical velocity the wake is absent.

At large values of Fr the turbulent agitation and the impulsive forces acting on the loose boundary grains appear to dominate the problem. The predicted tendency of the surface wave to be slightly upstream of the bed wave—the lower boundary—at subcritical flows can be observed in experiment. It is also in keeping with the calculated shift of maximum shear stress in flow over a wavy bed (Benjamin 1959).

Thus the varying shear stress introduced into an analytical model must be evaluated along the actual boundary to the flow, which in the case of a strong wake will include only a little of the sand bed. It may be seen from figure 4 that the turbulence shear over the wake is of greater magnitude than the temporal mean boundary shear over the crest region of the bed form—in keeping with the oscillating shear-stress concept. The variation of $\partial u/\partial y$ (vertical gradient of the mean flow) with x in the main flow is also seen to be appreciable (Raudkivi 1963).

The above over-simplified model yields a solution for the relative phase relationship that is in good agreement with observations in laboratory flumes. It does not require the assumption of an unknown phase lag in sediment movement as is necessary in potential-flow models and avoids the conclusion obtained from the constant-friction-factor model that the bed features will erode for both $Fr \lessgtr 1$ (cf. Reynolds 1965).

The deductions of the above model should be applicable also to wider flumes and rivers. However, the three-dimensional nature of the flow makes the problem much more complex. In narrow laboratory flumes the ground roller which approximates to a vortex filament, extends from wall to wall and straight-crested ripples and dunes are formed. Ripples formed in a wider flume have an irregular crest pattern, tending to a triangular pattern or tetrahedral form. The reason for this may be either that the vortex filaments are deflected and carried downstream as trailing vortices or they bind to the ground like loops or horseshoes. With increasing velocities the intensity and spacing of these vortices increases. The transverse parts are less subject to small disturbances and longer straight crests are formed as observed for dunes. The size of the bed forms depends on the size of the flow system. The relatively small ripple shown on figure 3 has a height of about one-fifth of the depth of flow. A corresponding bed form in a deeper flow would be of larger size. The regularity of the bed forms in plan and wavelength depends largely on the intensity of turbulence and on the large perturbations of the flow. Here the sand waves observed under low-turbulence tidal flows in very deep water are an extreme example. These have long straight crests, up to 40 miles have been reported, and are quite enormous in size, heights up to 60 ft. and wavelengths up to 3000 ft. However, relative to the depth of flow (50–100 fathoms) the maximum height to depth ratio is one-fifth to one-tenth.

The general form of variation of the average resistance factor with mean velocity and with shear velocity is known (Raudkivi 1963). The resistance factor increases from the initial flat-bed value as the velocity increases and bed forms develop, reaches a maximum and decreases to approximately the initial flat-bed value at transition flat bed.

The maximum value of the resistance factor corresponds to the maximum steepness of the bed waves. During the decrease of resistance the bed forms flatten, although they may initially grow in size, and the shear velocity changes little and may even decrease. This does not mean that the rate of bed load transport remains constant or decreases with increasing velocity. The total drag is composed of form and surface drag. Some of the turbulence generated by the form drag diffuses into the flow and decays, the rest contributes to the agitation of grains from the reattachment point downstream. The latter part together with the temporal mean surface drag effects the bed load transport. In order to describe sediment transport analytically, the contribution of form drag to agitation and transport has to be expressed in terms of flow parameters. This in turn will enable the determination of wavelength of the bed features.

It appears too that the hitherto unexplained phenomenon of ripples superimposed on the dune forms can be linked with the development of the boundary layer which starts at the reattachment point.

4. Description of experimental apparatus

The experimental work was carried out in a closed-circuit flume with a Perspex section 8 ft. long, 3 in. wide and 6 in. deep. It has provision for a 5 ft. long and 1·5 in. deep sand bed. The slope of the flume can be adjusted by a fine screw jack. The water and sand are circulated by a jet pump system and the rate of flow is measured by a venturimeter. On re-entering the flume the water is guided through a diffuser section and through honeycomb flow straighteners. The arrangement was arrived at by trial and error and yielded a logarithmic velocity distribution over a flat bed. Measurements of velocity and boundary shear distribution were made along the working section of the flume and aimed at ascertaining the effect of boundary-layer development. However, measurements at 3 ft. and 7 ft. sec-

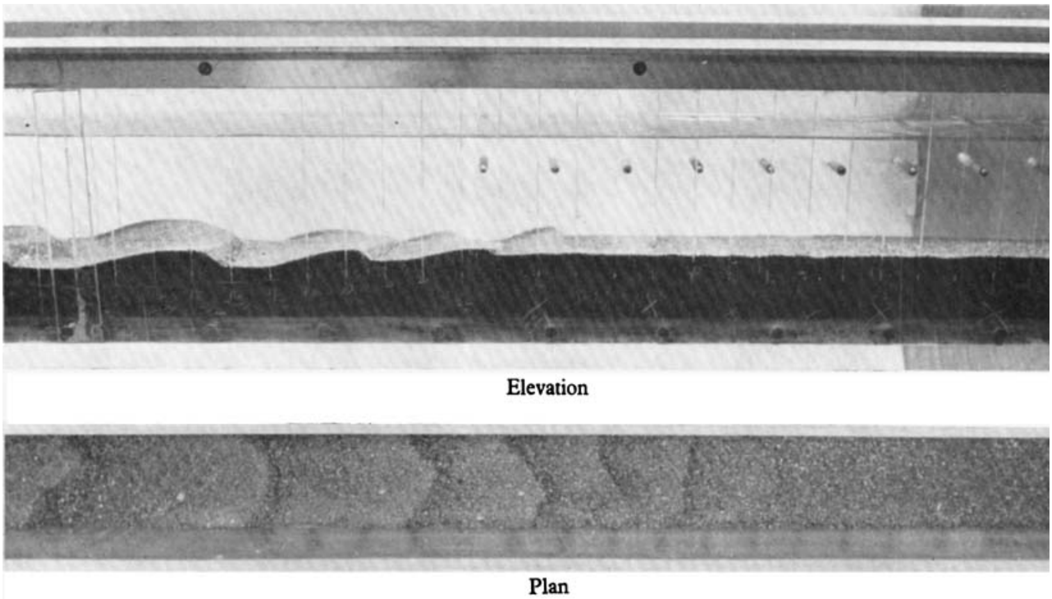

FIGURE 1. Elevation and plan view showing the propagating front and growth of bed forms.

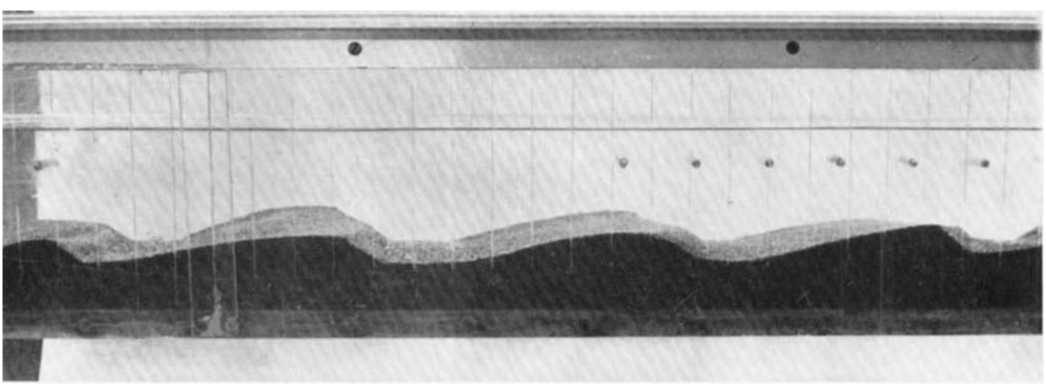

FIGURE 2. The steady-state pattern of bed waves obtained from the disturbance shown on figure 1.

tions yielded results which did not differ more than results from repeated measurements at the same station. This indicates that the level of turbulence intensity in this recirculatory system was such that the effect of changes in the boundary layer on the uniformity of flow were not obtrusive. The temperature was maintained constant by a thermostat-controlled heater and iced-water cooling coil. A Pitot tube was used for velocity measurements and two sizes of Preston tubes were used for surface-drag measurements. The pressure differences were measured with a Van Essen differential manometer and later with a fused-quartz pressure gauge, made by Texas Instrument Company. The turbulence measurements were carried out by a Lintronic hot-film anemometer. The measurement of turbulence in water is considerably more difficult than in air and the results are more prone to errors. The major sources of error are variations of the water temperature and the accumulation of impurities on the film of the probe. However, it was possible to reproduce the results in repeated experiments. Generally, the results are similar to those obtained in air but may be in error quantitatively by an instrument-induced factor.

Most of the experimental work has been described in more detail in a previous paper (Raudkivi 1963). However, the measurements of $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$ shown on figure 4 were obtained later, and are complementary to the measurements in the lee of a negative step described in the above reference. On comparison it is immediately apparent that the results of turbulence measurements over a fixed ripple form and in the lee of a negative step are similar. The turbulence measurements were made over a fixed ripple form but the results should be valid for a moving form because the rate of sediment transport in the original loose-boundary case was very small and only one grain layer in thickness (geometric mean diameter of grains = 0·40 mm). Furthermore, the turbulence measurements stopped at approximately 15 grain diameters from the bed surface, because of the instrument size. Therefore, it would be assumed that the moving grains will not modify the results significantly. The movement of the form, at 1·1 in./h, was so slow as to be stationary for the purpose of these measurements. However, the lower observations are affected by the probe-induced interference and are therefore not quantitatively reliable.

The noteworthy qualitative features of these turbulence measurements are the lateral diffusion and decay with distance downstream, particularly the rapid decrease in magnitude beyond the reattachment point. The turbulence energy determination is very inaccurate but the maximum is reached in the vicinity of the reattachment point and decreases from thereon to a level which is the same over all the crests. In general, generation of turbulence is predominant before the reattachment point is reached and spreading and dissipation thereafter. The bulk of the fluctuations have a frequency less than 50 cyc./sec.

5. Conclusion

The next step towards the solution of the sand-wave problem depends on linking the shear-stress distribution over the wake part of the main fluid flow boundary, and the shape of this wake, with flow parameters. There are essentially three problems involved simultaneously; the relationship between the particle size and flow parameters for conditions leading to wake formation; the contribution of form drag to sediment transport; and the distribution of shear stress over the lower boundary of the main fluid stream. It should be observed that with ripples and dunes the trough region of the shear-stress wave acts upon the sand bed, the peaks being over the wakes. With rapid sediment movement, near and beyond the transition flat-bed condition, a wavy sand boundary results and the oscillating departures from the hydrostatic-pressure distribution, together with the variation of shear stress over it, maintains the form. These remarks refer to a steady-state condition.

The development of the bed forms from a flat bed is a progressive process and not a spontaneous growth of amplitude of a bed disturbance.

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